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# Noncommutative $N = 2$ strings

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Abstract: We analyze open and mixed sector tree-level amplitudes of $N = 2$ strings in a space-time with $(2, 2)$ signature, in the presence of constant $B$ field. The expected topological nature of string amplitudes in the open sector is shown to impose nontrivial constraints on the corresponding noncommutative field theory. In the mixed sector, we first compute a 3-point function and show that the corresponding field theory is written in terms of a generalized $*$-product. We also analyze a 4-point function ($A_{oooc}$) of the mixed sector in $\Theta \to \infty$ limit.

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String Theories with $N = 2$ worldsheet supersymmetry [1] have been an important area of research [2]–[6] due to their connection with self-dual gravity and Yang-Mills. Such string theories live on a Kähler manifold with $(2,2)$ signature and their tree amplitudes have a 'magical' [2] property that the $n$-point functions are either local, or zero (for $n \geq 4$), thus having no 'effective' propagating degrees of freedom. These theories being intimately related to M(atrix) and F-theories [3, 4], are hence considered important from the point of view of obtaining the nonperturbative fundamental theory as well.

In this paper we study $N = 2$ strings in constant NS-NS antisymmetric tensor $(B)$ background, in view of interesting developments in noncommutative string theory [7]–[10]. In this regard, we have also been motivated by the fact that noncommutative $N = 2$ strings are expected to have interesting implications in possible generalizations of M(atrix) and F theories to include noncommutativity.

It is known that antisymmetric tensor backgrounds can be incorporated in $N = 2$ superspace formalism using chiral and twisted-chiral superfields [11]. In this manner, one has an $N = 2$ worldsheet supersymmetry without having a Kähler metric [11]. One now obtains a noncommutative complex manifold as the target space geometry.

There are two main highlights of the $N = 2$ noncommutative field theory obtained in this paper. The first is a nontrivial constraint, satisfied by the noncommutative theory, originating from the requirement of the absence of poles in the 4-point amplitude in the open sector of the $N = 2$ strings for nonzero $B$. The second is in the mixed sector. Here the noncommutative field theory involves a generalized $*$-product [12, 13], and $B$ explicitly appears with the open string metric in two (left/right) linear combinations for contracting the target space indices of the open-string fields. We finally analyze a 4-point mixed sector amplitude in the extreme noncommutative limit. Nontriviality in the computation of the tree-level string amplitudes in the mixed sector, stems from the fact that due to the absence of $z \to \bar{z}$-symmetry in the presence of $B$, one can not use the generalized Koba-Nielson integrals of [2, 5, 14] which are relevant when the domain of integration is the full complex plane.

One can consider $N = 2$ string action (in presence of $B$), written in an $N = 1$ superspace notation in [11]:

$$S = \int d^2x \int d\theta_L d\theta_R \left[ g_{IJ} D^\alpha X^I D_\alpha X^J + B_{IJ} D^\alpha X^I (\gamma^5 D)_\alpha X^J \right], \tag{1}$$

where the superspace field $X^I$ $(I \equiv 1, 2, \bar{1}, \bar{2})$ represents $X, Y, \bar{X}, \bar{Y}$, and $\alpha \equiv L, R$. The closed string metric and the antisymmetric background fields are denoted by $g_{IJ}$ and $B_{IJ}$ respectively.

Field equations, boundary conditions and canonical commutation relations, including those for fermions in $N = 2$ case turn out to be similar to the ones written in [15, 8]. Since the vacuum energy of the bosonic and fermionic oscillators remains



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same as for $B = 0$ [8], the spectrum of the theory once again consists of a scalar ($\varphi$) in the open and ($\phi$) closed string sector. The closed- [2] and open-string [5] vertex operators are given by:

$$
\begin{aligned}
V_o|_{\theta=0} &= e^{i(k\cdot\bar{x}+\bar{k}\cdot x)}\,, \qquad V_c|_{\theta=\bar{\theta}=0} = e^{i(k\cdot\bar{x}+\bar{k}\cdot x)}\,,\\
V_c^{\rm int} &= \left(ik\cdot\partial\bar{x} - i\bar{k}\partial x - k\cdot\bar{\psi}_R\bar{k}\cdot\psi_R\right)\left(ik\cdot\bar{\partial}\bar{x} - i\bar{k}\bar{\partial}x - k\cdot\bar{\psi}_L\bar{k}\cdot\psi_L\right)e^{i(k\cdot\bar{x}+\bar{k}\cdot x)}\,,\\
V_o^{\rm int} &= \left(ik\cdot\partial_\tau\bar{x} - i\bar{k}\cdot\partial_\tau x - k\cdot(\bar{\psi}_L+\bar{\psi}_R)\bar{k}\cdot(\psi_L+\psi_R)\right)e^{i(k\cdot\bar{x}+\bar{k}\cdot x)}\,,
\end{aligned}
\tag{2}
$$

where $V_{c,o}^{\rm int}$ are the closed- and open-string vertex operators that have been integrated w.r.t. their fermionic supercoordinates. Following [1], $\theta_L$ is set equal to $\theta_R$ for $V_o^{\rm int}$. Also, the bosonic component $x^i$, $x^{\bar{i}}$ denote $(x, y)$ and $(\bar{x}, \bar{y})$ which originate from (anti-)chiral and (anti-)twisted chiral fields of $N = 2$.

The two-point function for both bosons and fermions appearing in (2) can be written together using the superspace 2-point function in $N = 1$ notation of [16] (with $\alpha' = 1/2\pi$):

$$
\begin{aligned}
\langle X^i(Z_1,\bar{Z}_1)X^{\bar{j}}(Z_2,\bar{Z}_2)\rangle = &-g^{i\bar{j}}\ln\left[(z_1-z_2-\theta_1^L\theta_2^L)(\bar{z}_1-\bar{z}_2-\theta_1^R\theta_2^R)\right] +\\
&+(g^{i\bar{j}}-2G^{i\bar{j}})\ln\left[(z_1-\bar{z}_2-\theta_1^L\theta_2^R)(\bar{z}_1-z_2-\theta_1^R\theta_2^L)\right] -\\
&-2\Theta^{i\bar{j}}\ln\left[\frac{z_1-\bar{z}_2-\theta_1^L\theta_2^R}{\bar{z}_1-z_2-\theta_1^R\theta_2^L}\right].
\end{aligned}
\tag{3}
$$

The indices $i, \bar{j}$ run over $1, 2$ and $\bar{1}, \bar{2}$ respectively. The open string metric $G^{i\bar{j}}$ and the noncommutativity parameter $\Theta^{i\bar{j}}$ can be expressed in terms of $g^{i\bar{j}}$ and $B^{i\bar{j}}$ as in [8]. For our case, $g^{i\bar{j}}$ denotes the flat closed string metric and $B^{i\bar{j}}$, constant antisymmetric background of 'magnetic' type.

Now, using the above results, we calculate various string amplitudes in the open and mixed sectors. In the closed sector the results of [2, 5] are still valid as closed strings have no boundary, and hence are insensitive to the addition of boundary terms to the world-sheet action. In the open- and mixed-string sectors, the super-Möbius transformations allow two complex fermionic supercoordinates to be set to zero, and three real bosonic coordinates to be fixed to any arbitrary value.

**Open sector.** The 3-point function using obvious notations is given by:

$$
\begin{aligned}
A_{ooo}(B\neq 0) &= \langle V_o|_{\theta=0}(0)V_o^{\rm int}(1)V_o(\infty)|_{\theta=0}\rangle\\
&= e^{\frac{i}{2}(\bar{k}_1\Theta k_2-\bar{k}_2\Theta k_1)}A_{ooo}(B=0)\,,
\end{aligned}
\tag{4}
$$

where $A_{ooo}(B = 0) = c_{12} \equiv k_1G^{-1}\bar{k}_2 - k_2G^{-1}\bar{k}_1$. Now, as in [5], one has to impose Bose symmetry on $A_{ooo}$ in (4). Unlike [5], for the noncommutative case, one can have an "isoscalar" as well as an "isovector" component of the amplitude:

$$
A_{ooo} = A_{ooo}^{\rm S} + A_{ooo}^{\rm AS\ abc}\,,
\tag{5}
$$

where $A^{\rm S}_{ooo}$ is the isoscalar part of the amplitude that is symmetric under the interchange of the momentum labels of particles 1 and 2, and $A^{\rm AS\ abc}_{ooo}$ is the isovector part of the amplitude that is antisymmetric under the interchange of the momentum and group labels separately, but is symmetric under the simultaneous interchange of both types of labels. Denoting $K_{12} \equiv \frac{1}{2}(\bar{k}_1\Theta k_2 - \bar{k}_2\Theta k_1)$ one sees that

$$A^{\rm S}_{ooo} = c_{12}\sin(K_{12})\,, \qquad A^{\rm AS\ abc}_{ooo} = c_{12}cos(K_{12})f^{abc}\,. \tag{6}$$

Similarly, the 4-point function is given by:

$$\begin{aligned} A_{oooo}(B\neq 0) &= \int_0^1 \langle V_0|_{\theta=0}(0)V_o^{\rm int}(x)V_o^{\rm int}(1)V_o|_{\theta=0}(\infty)\rangle \\ &= e^{i(K_{12}+K_{23}+K_{13})}A_{oooo}(B=0)\,, \end{aligned} \tag{7}$$

where $A_{oooo}(B=0) = F\frac{\Gamma(1-2s)\Gamma(1-2t)}{\Gamma(2u)}$ as in [5]. The $\Theta$-dependent phase factor in (7) matches with the phase factor in equation (2.11) of [8]. The null kinematic factor $F$ is that of [2] with the difference that the open-string metric is used for contracting momenta in $s, t, u$ as well as $c_{ab}$'s.

Moreover, since in the purely open string sector, the $\Theta$ dependence of the amplitude enters via a phase factor, one sees that the above result for the noncommutative 3- and 4-point functions readily generalizes to the noncommutative $n$-point function, implying that, like the claim for the commutative $N=2$ theory, all $n$-point functions with $n\geq 4$ also vanish. Hence, the noncommutative $N=2$ theory is "topological" in the closed- and open-string sectors.

We now analyze in some detail the implications of the above modifications to the field theory of open string scalars. Using (5) and (6), one can evaluate the field theory (FT) amplitude $A_{oooo\ FT}$ which consists of contributions from two 3-point functions ($A_{ooo\ FT}$'s) as well as a contact vertex $V_{oooo\ FT}$ (whose form is determined from the requirement that $A_{oooo\ FT}$ like $A_{oooo}$ of string theory, vanishes). One can verify that for $A_{ooo\ FT}$ corresponding to $A^S_{ooo}$ in (6), there are no poles in $A_{oooo\ FT}$. This can be seen by adding contributions to the 4-point amplitudes from $s$, $t$ and $u$-channels as:

$$A^S_{ooooFT} = A\sin(K_{12})\sin(K_{34}) + B\sin(K_{23})\sin(K_{41}) + C\sin(K_{31})\sin(K_{24})\,, \tag{8}$$

where

$$A = \frac{c_{12}c_{34}}{s}\,, \qquad B = \frac{c_{23}c_{41}}{t} = u - A\,, \qquad C = \frac{c_{31}c_{24}}{u} = t + A\,. \tag{9}$$

Then by eliminating $k_4$ using momentum conservation, it is noticed that pole part of the amplitude above cancels for $A^S_{ooo}$ in equation (8). In other words, $\sin(K_{ab})$ in $A^S_{ooooFT}$ acts as a structure constant. To generalize this result further, one can consider a more general 3-point function

$$A^{S\ abc}_{ooo} = c_{12}\sin(K_{12})d^{abc}\,, \tag{10}$$

with $d^{abc}$ being symmetric structure constants. Vanishing of poles in $A^S_{oooo\ FT}$ then implies a strong condition on $d^{abc}$'s leading to multiple copies of abelian noncommutative FT's mentioned in (14) below.

For $A^{AS}_{ooo}$, on the other hand, we get a constraint on $f^{abc}$:

$$\begin{aligned}&\cos(K_{12})\sin(K_{31})\sin(K_{32})f^{abx}f^{xbd}-\\&-\cos(K_{23})\sin(K_{21})\sin(K_{31})f^{bcx}f^{xda}+\\&+\cos(K_{31})\sin(K_{21})\sin(K_{23})f^{cax}f^{xbd}=0\,.\end{aligned} \tag{11}$$

One sees that the above constraint can not be satisfied by any classical group. Perhaps it may be satisfied for some quantum group. One now observes that U($N$) gauge groups can be obtained from 3-point string vertex, eq. (4), by considering mixed (isoscalar-isovector) vertices. In particular, for U(2), after imposing Bose symmetry on two of the external legs in eq. (4), the corresponding isoscalar-isovector field theory vertex is given as:

$$A^{ab}_{\rm Mixed}=c_{12}\sin(K_{12})\delta^{ab}\,. \tag{12}$$

Then it can once again be shown that the poles in the isovector 4-point amplitude, obtained by sewing together two 3-point vertices with isoscalar and isovector internal states, cancel.[1] Higher rank groups can also be incorporated by including 3-point vertex in eq. (10) (See for example [17]).

We now write down the FT corresponding to $A^S_{ooo\ FT}$ mentioned before, as well as the contact vertex appearing in equation (8), after using equation (9) whose explicit form is

$$V^{\rm int}_{oooo}=u\sin(K_{23})\sin(K_{41})+t\sin(K_{31})\sin(K_{24})\,. \tag{13}$$

One then obtains the field theory action corresponding to $A^S_{ooo}$ and $V^{int}_{oooo}$ up to terms quartic in $\varphi$:

$$\mathcal{L}_{FT}=G^{\bar{i}j}\left[\frac{1}{2}\bar{\partial}_{\bar{i}}\varphi*\partial_j\varphi+\frac{i}{3}[\bar{\partial}_{\bar{i}}\varphi,\partial_j\varphi]_* *\varphi-\frac{1}{12}\bar{\partial}_{\bar{i}}\varphi*[[\partial_j\varphi,\varphi]_*,\varphi]_*\right], \tag{14}$$

where $[\xi,\eta]_*\equiv\xi*\eta-\eta*\xi$. The $*$-product is defined (in momentum space) as:

$$e^{ik_1\cdot x}*e^{ik_2\cdot x}=e^{iK_{12}}e^{i(k_1+k_2)\cdot x}\,. \tag{15}$$

with $k\cdot x\equiv k\cdot\bar{x}+\bar{k}\cdot x$. A generalization of the above noncommutative abelian field theory action to U($N$) case is straightforward. The Moyal deformation of self dual Yang Mills (and gravity) were also considered in [18]. We now study the mixed sector of the noncommutative $N$=2 theory.

[1] We thank the referee to point this out.

**Mixed sector.** ($a$) $A_{ooc}$: we now show, in $N=2$ context, that a mixed amplitude with two open and one closed string vertices generates a field theory with a generalized $*$-product [12, 13] (the $\Theta=0$ limit of which reduces to the result of [5]).

Following [5], for the purpose of setting the limits of integration, it is convenient to fix the bosonic coordinate of one of the two $V_o$'s (to 0) and that of $V_c$ to $z=x+iy$. Then

$$
\begin{aligned}
A_{ooc} &= \int_{-\infty}^{\infty} db \langle V_o|_{\theta=0}(b) V_o^{\rm int}(z=x+iy) V_o|_{\theta=0}(\tau\to\infty)\rangle \\
&= e^{\frac{i}{2}(\bar{k}_b\Theta k_\tau - \bar{k}_\tau\Theta k_b)} \times \\
&\quad\times \frac{y}{4\pi^2}\int_{-\infty}^{\infty} db\left(c_{b\tau}^2 - (k_\tau\Theta\bar{k}_b + k_b\Theta\bar{k}_\tau)^2\right)\frac{e^{-\frac{1}{2\pi}(k_b\Theta\bar{k}_\tau - k_\tau\Theta\bar{k}_b)\ln\left(\frac{b-(x-iy)}{b-(x+iy)}\right)}}{([b-x]^2+y^2)} \\
&= \frac{e^{\frac{i}{2}(\bar{k}_b\Theta k_\tau - \bar{k}_\tau\Theta k_b)}}{4\pi^2}\left(c_{b\tau}^2 - (k_\tau\Theta\bar{k}_b + k_b\Theta\bar{k}_\tau)^2\right)\pi e^{-i\frac{(k_b\Theta\bar{k}_\tau - k_\tau\Theta\bar{k}_b)}{2}}\frac{\sin\left(\frac{k_b\Theta\bar{k}_\tau - k_\tau\Theta\bar{k}_b}{2}\right)}{(k_b\Theta\bar{k}_\tau - k_\tau\Theta\bar{k}_b)} \\
&= \frac{\pi}{2}c_{b\tau}^L c_{b\tau}^R\frac{\sin(K_{b\tau})}{K_{b\tau}},
\end{aligned}
\tag{16}
$$

where $c_{ab}^{L,R} \equiv \frac{1}{2\pi}(k_a G^{-1}\bar{k}_b - k_b G^{-1}\bar{k}_a \mp k_a\Theta\bar{k}_b \pm k_b\Theta\bar{k}_a)$, the upper and lower signs corresponding to $L$ and $R$ respectively. The $b$ integral above was done using Mathematica and is also given in appendix A of [19].

Before commenting on the topological nature of this amplitude, we now write down the corresponding interaction term in the FT action which is given in terms of a generalized $*$-product [12, 13]:

$$
\mathcal{L}_{ooc} = \phi\left(\partial_i\partial_j\varphi *' \bar{\partial}_{\bar{i}}\bar{\partial}_{\bar{j}}\varphi - \partial_i\bar{\partial}_{\bar{i}}\varphi *' \partial_j\bar{\partial}_{\bar{j}}\varphi\right)(G^{-1}-\Theta)^{i\bar{j}}(G^{-1}+\Theta)^{j\bar{i}}, \tag{17}
$$

where we have made use of

$$
e^{ik_1\cdot x} *' e^{ik_2\cdot x} = \frac{\sin(K_{12})}{K_{12}} e^{i(k_1+k_2)\cdot x}. \tag{18}
$$

in arriving at (17) from (16).

Now, to interpret $A_{ooc}$ as a topological theory in the sense of [2], one now sees that by expanding $\sin(K_{12})/K_{12}$ in a power series in $\Theta$, an infinite number of ($\Theta$ or equivalently $B$-dependent) terms are generated at the 3-point level in the mixed sector. As the radius of convergence of the $\sin x/x$ expansion is infinite, this implies that *after expansion*, $A_{ooc}$ can be interpreted as an infinite series of local interactions between the closed and open string scalars.

($b$) $A_{oooc}$: like [5], we set the bosonic coordinates of the three $V_o$'s at $0, 1, \infty$ and the fermionic coordinates of the first and third $V_o$'s to zero. We will hence require to integrate over the bosonic coordinates of $V_c^{\rm int}(z,\bar{z})$. Defining $t^{L,R} \equiv \frac{1}{2\pi}(k_1 G^{-1}\bar{k}_4 + k_4 G^{-1}\bar{k}_1 \mp k_1\Theta\bar{k}_4 \mp k_4\Theta\bar{k}_1)$, $u^{L,R} \equiv \frac{1}{2\pi}(k_1 G^{-1}\bar{k}_3 + k_3 G^{-1}\bar{k}_1 \mp k_1\Theta\bar{k}_3 \mp k_3\Theta\bar{k}_1)$, one gets:

$$
\begin{aligned}
A_{oooc} &= \iint_{\rm UHP} dz d\bar{z} \langle V_o|_{\theta=0}(0) V_o^{\rm int}(1) V_o|_{\theta=0}(\infty) V_c^{\rm int}(z,\bar{z})\rangle \\
&\sim \iint_{\rm UHP} dz d\bar{z} z^{t^L} \bar{z}^{t^R} (1-z)^{u^L} (1-\bar{z})^{u^R} \times \\
&\qquad \times \Bigg( \left[-\frac{c_{14}^L}{z} + \frac{c_{24}^L}{(1-z)}\right] \left[-\frac{c_{14}^R}{\bar{z}} + \frac{c_{24}^R}{(1-\bar{z})}\right] \left[-2c_{12} + \frac{c_{24}^L}{(1-z)} + \frac{c_{24}^R}{(1-\bar{z})}\right] + \\
&\qquad + \left[-\frac{c_{14}^L}{z} + \frac{c_{24}^L}{(1-z)}\right] \left[\frac{-u^R + (u^R)^2 - (c_{24}^R)^2}{(1-\bar{z})^2}\right] + \\
&\qquad + \left[-\frac{c_{14}^R}{\bar{z}} + \frac{c_{24}^R}{(1-\bar{z})}\right] \left[\frac{-u^L + (u^L)^2 - (c_{24}^L)^2}{(1-z)^2}\right] \Bigg),
\end{aligned}
\tag{19}
$$

which gives the $A_{oooc}$ amplitude of [5] for $\Theta = 0$. However, because of the lack of $z \to \bar{z}$ symmetry in the presence of $B$, unlike [5], one can not enlarge the domain of integration from the upper half complex plane to the entire complex plane

To simplify the algebra, one notices that in the extreme noncommutative case ($\Theta \to \infty$), which has been a subject of recent interest [9], $t^L = -t^R = \tilde{t}$, $u^L = -u^R = \tilde{u}$ and $c_{ab}^L = -c_{ab}^R = \tilde{c}_{ab}$. We will now consider only this case in the paper. One can then verify that the coefficient of the leading term vanishes when one makes use of the fact that any finite powers of $z, \bar{z}, (1-z), (1-\bar{z})$ can be dropped with respect to the ones containing $t^{L,R}, u^{L,R}$. This suggests that 4-point amplitude of (19) is possibly zero. We now argue that each of the integrals appearing in eq. (19) is in fact zero for generic non-integral $\tilde{t}$ and $\tilde{u}$, positive integral values of $\tilde{t}$ or $\tilde{u}$, as well as negative integral values of $\tilde{u}, \tilde{t} < -4$. We hence have to evaluate integrals of the type

$$
\int\int dz d\bar{z} z^a (1-z)^b \bar{z}^c (1-\bar{z})^d \tag{20}
$$

over the UHP. The above integral is similar, though not identical to the ones that are evaluated in [19].

Using the Stokes theorem in the complex plane, the integral of (20) gets mapped to

$$
\left(\int_{\mathrm{Im}\, z=0 \ \text{axis}} + \int_{C_R}\right) \frac{\bar{z}^c (1-\bar{z})^d z^{1+a}}{1+a} \ {}_2F_1(1+a, -b, a+2; z)\,, \tag{21}
$$

where $C_R$ is a semicircular contour in the upper half plane whose radius is taken to infinity eventually. Now, we use the integral representation of ${}_2F_1(1+a, -b, a+2; z)$ as given in [20, eq. (15.3.1)] valid for $a+2 > -b > 0$, $|\,\mathrm{Arg}(1-z)| < \pi$. The term $c_{12}$ in equation (19) drops out as compared to terms consisting of $c_{24}^L$ or $c_{24}^R$. From (19) one will pick up an $R^{-3}$ from each of the terms. So, when evaluating $\int_{C_R}$, one gets $\lim_{R\to\infty} \int_C \sim R^{c+d-2}$. Hence, one takes $c + d < 2$ for the purpose of evaluation of the integral so that $\lim_{R\to\infty} \int_C = 0$, keeping in mind that the answer that one gets

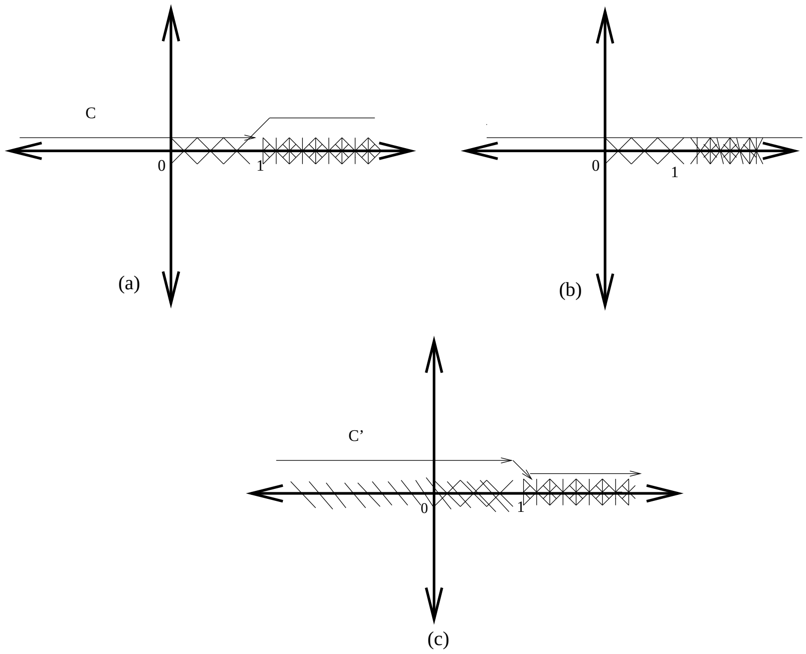


**Figure 1:** Deformation of contour of integration; there are three branch cuts: $x \to -\infty$ to $x = 1$, $x = 0$ to $x = 1$, and $x = 1$ to $x \to \infty$.

can be analytically continued to $c + d \geq 2$ domain, and those $a, b$ not satisfying the above constraints. Hence, one is left with the integral over the real axis which is:

$$\int_{-\infty}^{\infty} dx\, x^{a+c+1}(1-x)^d\, {}_2F_1(1+a, -b, a+2; x)\,. \tag{22}$$

If $b \in \mathbb{Z}^+$, then ${}_2F_1(1+a, -b, a+2; x)$ can be expanded as a finite series, and using that $\int_{-\infty}^{\infty} dx x^\alpha (1-x)^\beta$ is proportional to $\sin(\pi\alpha)$ (for more exact expression, see (24)), one sees that (22) vanishes. For $b \in \mathbb{Z}^-$ and $b < -4$, one performs the $\bar{z}$ integration first, and the argument for $b \in \mathbb{Z}^+$ follows here as well. For $a \in \mathbb{Z}$, we conformally map the UHP to the LHP by $z \to (1-z)$. Then the argument for $b \in \mathbb{Z}$ can be repeated here. So, (22) vanishes if at least one of $a$ or $b$ is a positive integer or a negative integer less than $-4$. We now argue that (22) vanishes for $a, b \notin \mathbb{Z}$ as well.

The identity (15.3.6) of [20] which is valid for $|\,\mathrm{Arg}(1-z)| < \pi$, is now used. The integrand in (22) is analytic in the entire complex plane except along the branch cuts from $x = 0$ to $x = 1$ and along $x > 1$. Hence, one can deform the contour along the real axis to a contour $C$ (which is equivalent to putting $z = x + i\epsilon$ for $x > 1$) of figure 1$a$. Along $C$, $|\,\mathrm{Arg}(1-z)| < \pi$ is satisfied for the entire contour. After applying (15.3.6) (of [20]) to (22), one deforms $C$ of figure 1$a$ back to the contour of figure 1$b$.

Now, the Mellin-Barnes contour integral representation of the hypergeometric function ${}_2F_1(\alpha, \beta, \gamma; (1-z))$ as given in (15.3.2) of [20] valid for $|\,\mathrm{Arg}[-(1-z)]| < \pi$, is used. Like before, one uses the analyticity property of the integrand of (22), and deforms the contour of figure 1$b$ to $C'$ (which is equivalent to setting $z = x + i\epsilon$ for $x < 1$) of figure 1$c$. Along $C'$, $|\,\mathrm{Arg}[-(1-z)]| < \pi$ is satisfied for the entire contour. Finally, we deform $C'$ back to figure 1$b$. We will thus have:

$$\int_{-\infty}^{\infty} dx x^{a+c+1}(1-x)^{d+s+\lambda}\,, \tag{23}$$

where $\lambda = 0$ or $1 + b$. The above integral, after evaluation, has a form:

$$\int_{-i\infty}^{i\infty} ds \frac{\sin(\pi[a+c+1])\sin(\pi[d+s+\lambda])}{\sin(\pi[a+c+d+s+\lambda+1])} \frac{\Gamma(a+c+2)\Gamma(d+s+\lambda+1)}{\Gamma(a+c+d+s+\lambda+3)}\,, \tag{24}$$

with certain restrictions on $a, c, d, \lambda$ which can then be removed by analytic continuation. In the large $B$ limit $a + c \in \mathbb{Z}$, hence $\sin(\pi[a + c + 1]) = 0$ implying that (22) vanishes. As a result, $A_{oooc}(B \to \infty) = 0$ in all the cases discussed above.

We end by pointing out that it will be interesting to examine the applications of $N = 2$ noncommutativity to M(atrix) and F- theories.

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